Stability Analysis of an Underground Opening

The Earth’s crust is under stress and can be described as a dynamic stress field. A portion of the stress was generated during the formation of the planet Earth, and another portion is generated due to gravitational interaction—predominantly between the Earth and the Moon, but also with other planets. The stress state of the Earth's crust is defined by three stress components: Vertical stress (σ_v); Maximum horizontal stress (σ_H); Minimum horizontal stress (σ_h). This condition is referred to as the initial stress state, undisturbed stress state, or virgin stress state. The excavation of an underground opening in such a rock mass forms a different stress pattern, and this is described as the induced stress state. Most underground structures can be reduced to a plane strain model, where all cross-sections perpendicular to the axis of the elongated opening are identical. In this case, the stress state is described using: Maximum principal stress (σ_1); Minimum principal stress (σ_3), which lie in the plane of the cross-section; and the stress perpendicular to the cross-section (σ_2).

The term “induced stresses” leads many colleagues to conclude that these are the stresses caused by the excavation of the opening. However, the stress in the rock mass is the potential energy of elastically deformed rock. By removing part of the rock, the particles at the boundary of the cavity, having lost support, move toward the void. This movement is opposed by the tensile strength of the rock (cohesion), internal friction resistance, and the structural support resistance. As a consequence of this disturbance, a new stress pattern emerges. For the assessment of the stability of the opening, stresses are not the key—deformations are, or more precisely, dilation. When the dilation exceeds a critical value, tensile failure occurs, a crack is formed, and the affected portion of the rock changes its structure. This change may or may not result in progressive failure.

Total stresses are not greater—they are the same. In fact, they are not the same but lower, because a portion of the deformation work is entropy—converted into heat. However, since we are interested in dilation, we observe the total deformation work. The redistribution of stress is influenced by the shape of the underground opening. The influence of shape is most easily established using 2D analysis—for example, by the finite element method. The influence of shape is manifested in the directions of the principal stresses and their intensity.

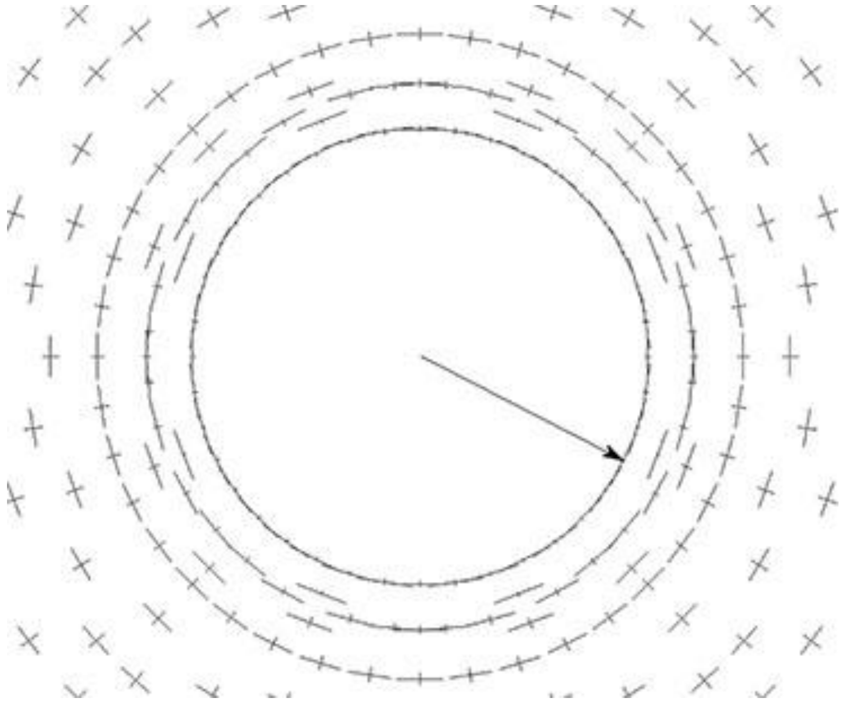

Figure 1. Directions of principal stresses

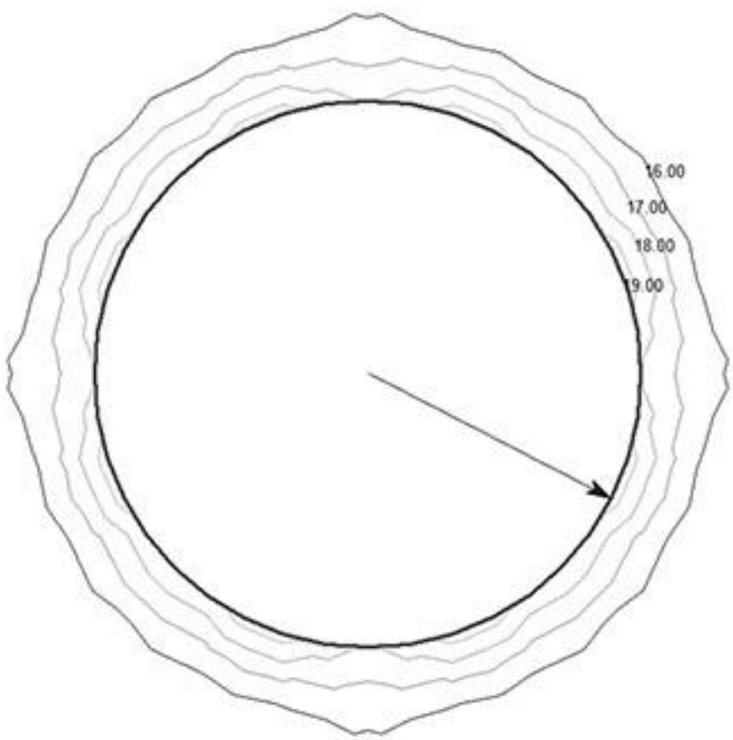

Figure 2. Intensity of maximum principal stresses

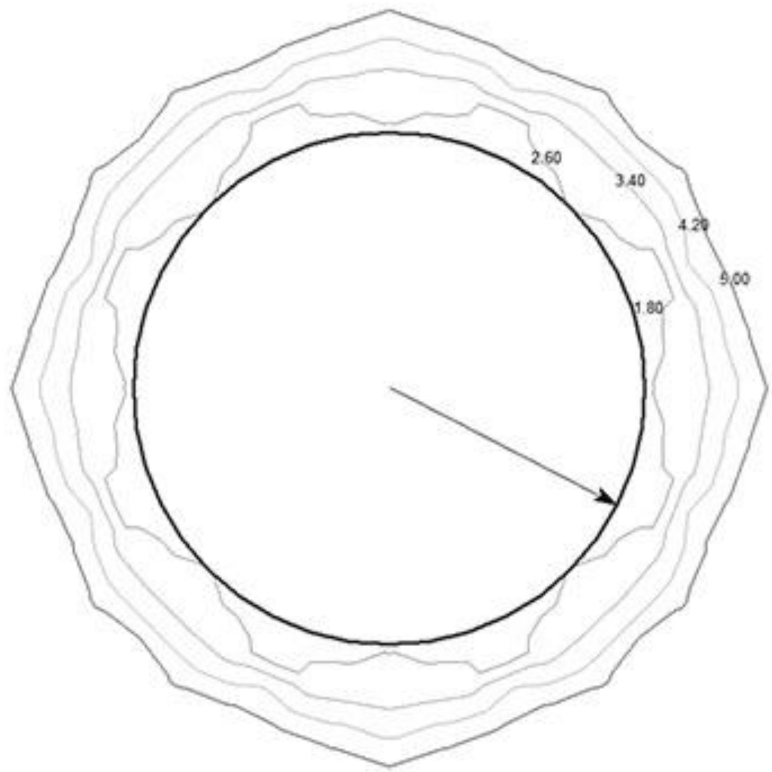

Figure 3. Intensity of minimum principal stresses

For an underground opening of circular cross-section excavated in a rock mass with an initial stress state of ($\sigma_1 = 10MPa$, $\sigma_2 = 10MPa, \sigma_3 = 10MPa$), the distribution of principal stresses appears as in Figure 1, showing directions of principal stresses. Figures 2 and 3 show the intensity of principal stresses.

As previously stated, the relations between components of the stress state are shown in Figure 4 and described by the equation:

$$\sigma_1 = \frac{\sigma_t}{\nu}(1 + tg\varphi) + \sigma_3$$

In the plane of tensile fracture lies the trajectory of the maximum principal stress, perpendicular to the direction of the minimum principal stress.

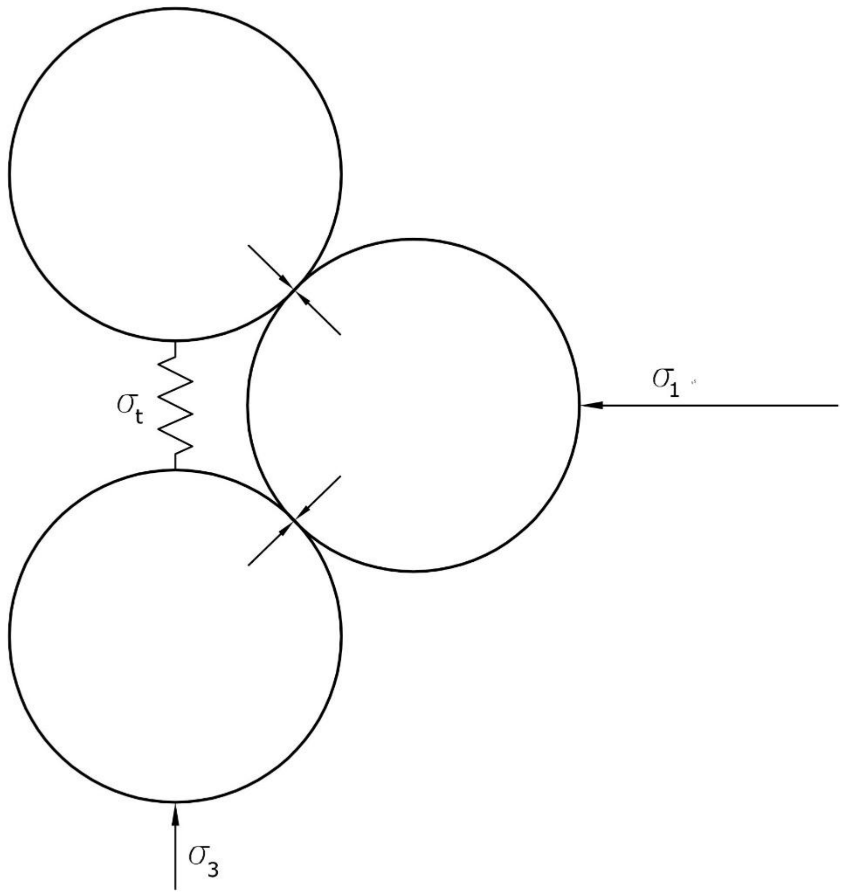

Figure 4. Relationships between stress components

This relation can be rewritten as:

$$\sigma_t = \frac{\nu}{1 + tg\varphi} \cdot \sigma_1 - \sigma_3$$

For the underground opening to be stable, tensile stresses should not exceed the tensile strength of the rock.

In the above equation, the term:

$$\frac{\nu}{1 + tg\varphi}$$

is a material property and indicates the strength of the rock, so we can write:

$$\sigma_t = st \cdot \sigma_1 - \sigma_3$$

Then:

$$st = \frac{0,25}{1 + tg40^{\circ}} = 0.14$$

So the value of the tensile stress will be:

$$\sigma_t = 0,14 \cdot 19 - 1,8 = 0,78MPa$$

This example analyzes a circular profile, as used in vertical shafts and horizontal or inclined tunnels excavated using the TBM method. In most cases, underground openings are constructed with an arched profile of varying curvature, or sometimes rectangular. This example was based on a hypothetical stress state where all three components are equal—a situation which does not occur in practice. This was used as a reference for subsequent real stress states.

In the next example, the same underground opening in the same rock is modeled for the case where the maximum principal stresses, whose trajectory is horizontal, are ($\sigma_1 = 10MPa$, $\sigma_2 = 5MPa, \sigma_3 = 5MPa$).

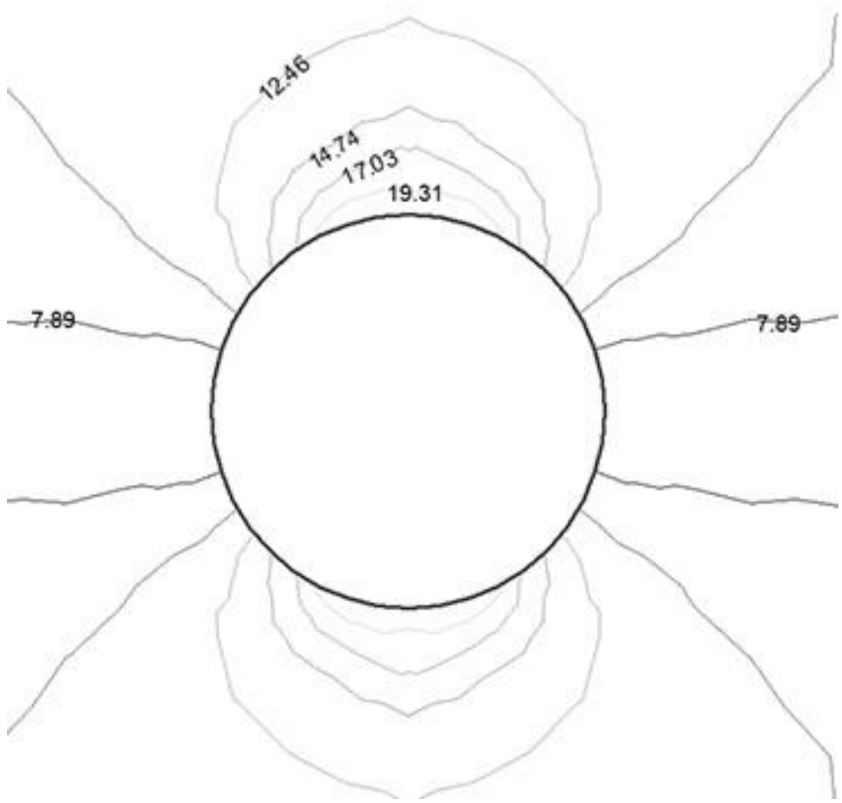

Figure 5. Intensity of maximum principal stresses

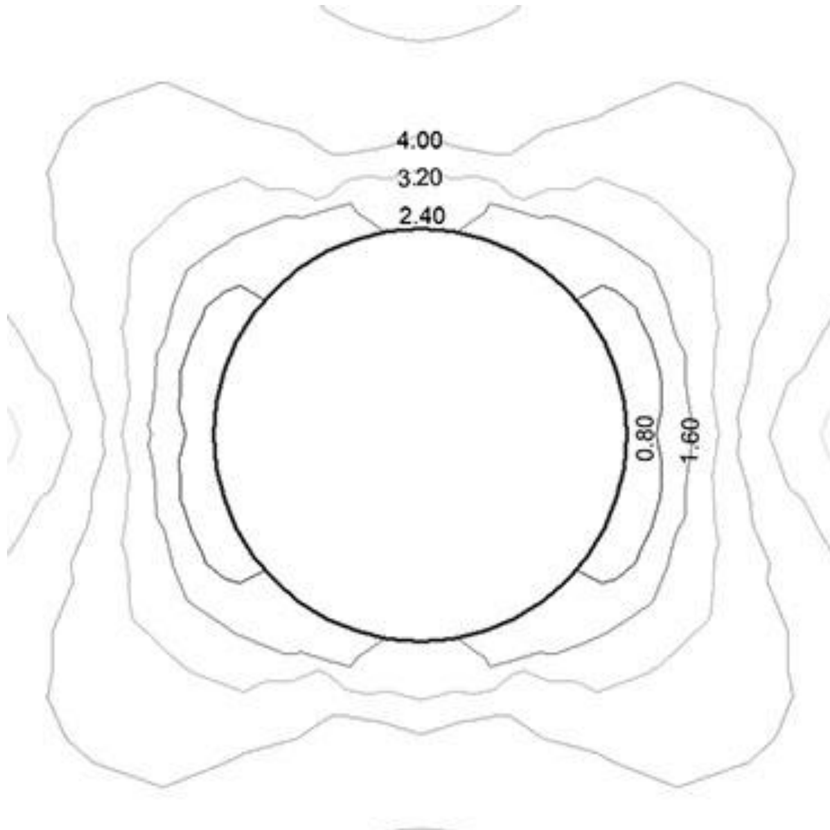

Figure 6. Intensity of minimum principal stresses

In these figures, we can observe that higher values of induced principal stresses occur in the floor and roof, suggesting that tensile stresses will be greater in those areas. However:

Induced tensile stress in the roof of the opening:

$$\sigma_t = 0{,}14 \cdot 19{,}31 - 2{,}4 = 0.3MPa$$

Induced tensile stress in the wall of the opening:

$$\sigma_t = 0{,}14 \cdot 7{,}89 - 0{,}8 = 0.3MPa$$

The tensile stresses that cause failure and instability are the same as in the previous case with equal initial stress components. The key factor is the shape of the underground opening. It is true that the absolute value of the tensile stress is lower because the total intensity of all stress components is lower. This holds under the assumption that the rock is isotropic and has the same Poisson's ratio and internal friction angle in all directions—which is often not the case, but rarely receives attention in practice.

The next example is an opening of square cross-section. Again, we show the intensity of the minimum and maximum principal stresses (Figures 7 and 8). The same rock strength parameters and initial stress state are assumed ($\sigma1$ = 10 MPa, $\sigma2$ = 5 MPa, $\sigma3$ = 5 MPa).

We now observe the same minimum values but different maximum values of principal stress in the walls and in the roof/floor.

Thus:

Induced tensile stress in the roof:

$$\sigma_t = 0{,}14 \cdot 13{,}33 - (-0{,}45) = 2{,}32MPa$$

Induced tensile stress in the wall:

$$\sigma_t = 0{,}14 \cdot 3{,}33 - (-0{,}45) = 0.92MPa$$

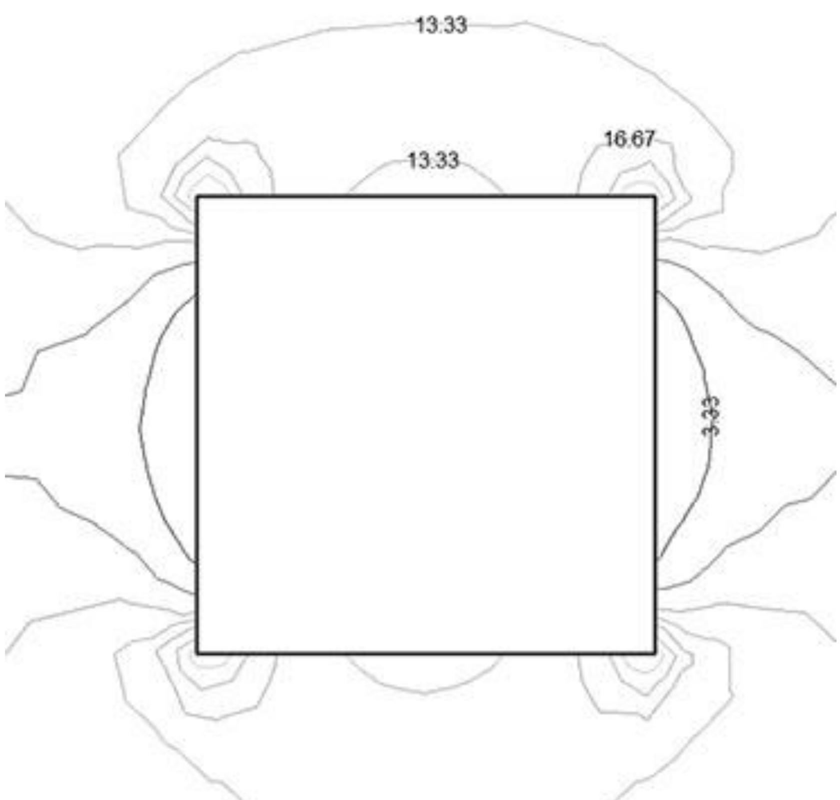

Figure 7. Intensity of maximum principal stresses

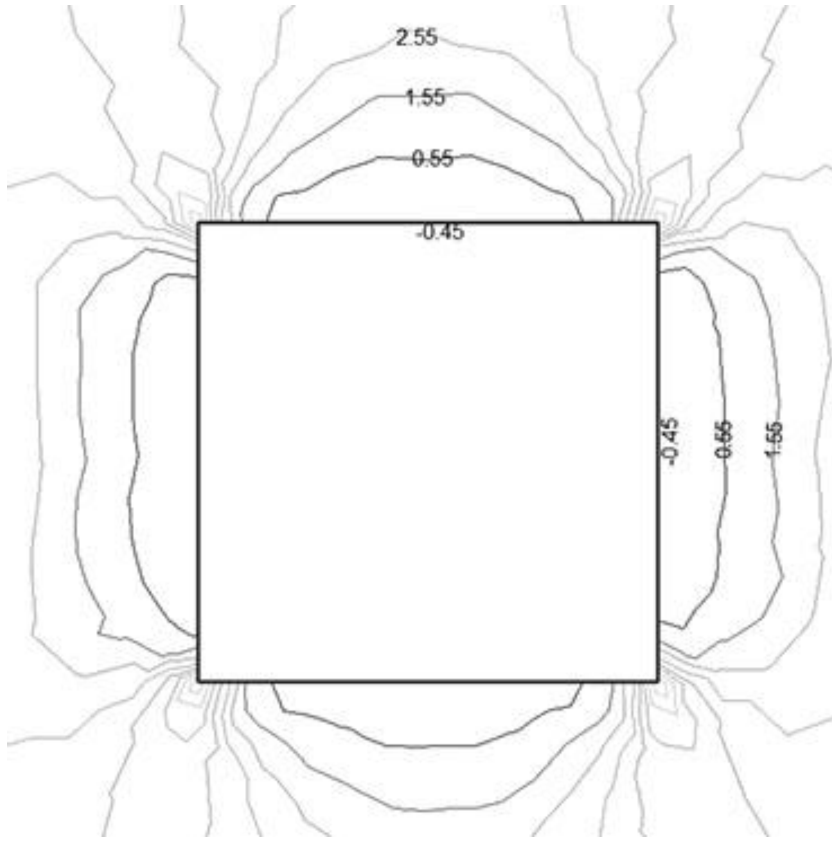

Figure 8. Intensity of minimum principal stresses

We observe that greater tensile stresses are induced in the roof and floor of the opening.

We have a significantly different situation in the corners of the opening. Figures 9 and 10 show the values of principal stresses with their directions—indicating significantly higher stress values. The situation is identical in all four corners.

So the induced tensile stress in the corners of the opening will be:

$$\sigma_t = 0{,}14 \cdot 23{,}33 - 7{,}55 = -6{,}96MPa$$

The negative sign indicates compressive stress.

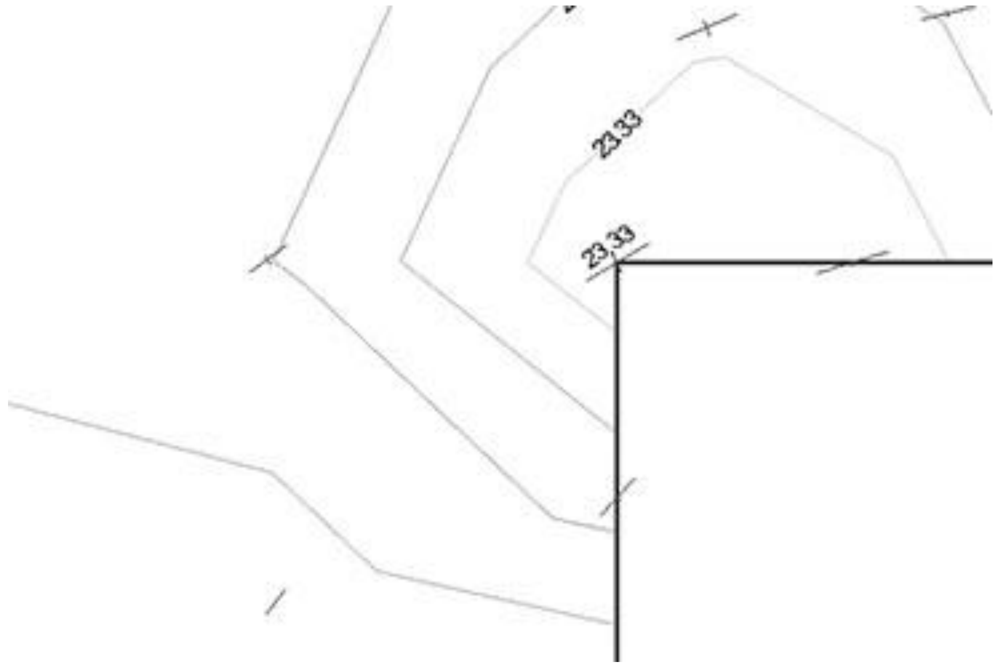

Figure 9. Intensity of maximum principal stresses

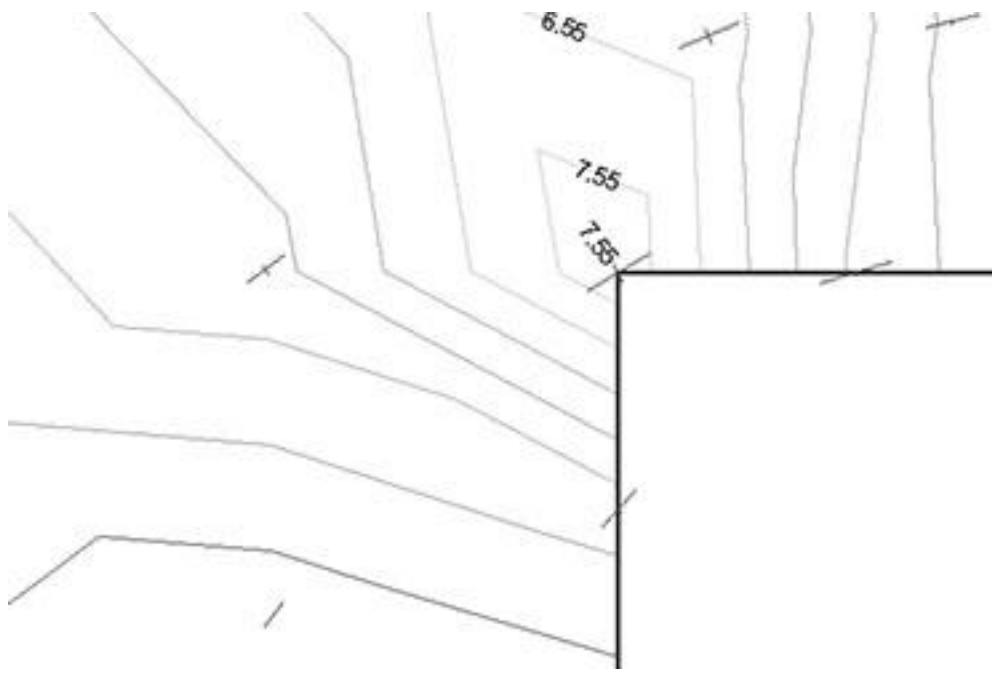

Figure 10. Intensity of minimum principal stresses

What happens in the corner of the underground opening is illustrated with circles symbolizing particles, as in earlier examples (Figure 11). As seen in Figures 9 and 10, in the corner, the direction of the minimum principal stress follows the bisector of the angle, and the maximum principal stress is perpendicular to it. The position of the particles forming the corner determines the intensity of minimum and maximum stresses. The equation ($\sigma_t = st \cdot \sigma_1 - \sigma_3$) has two terms: ($st \cdot \sigma_1$), which tends to induce tensile stress and failure, and (σ_3), which resists it.

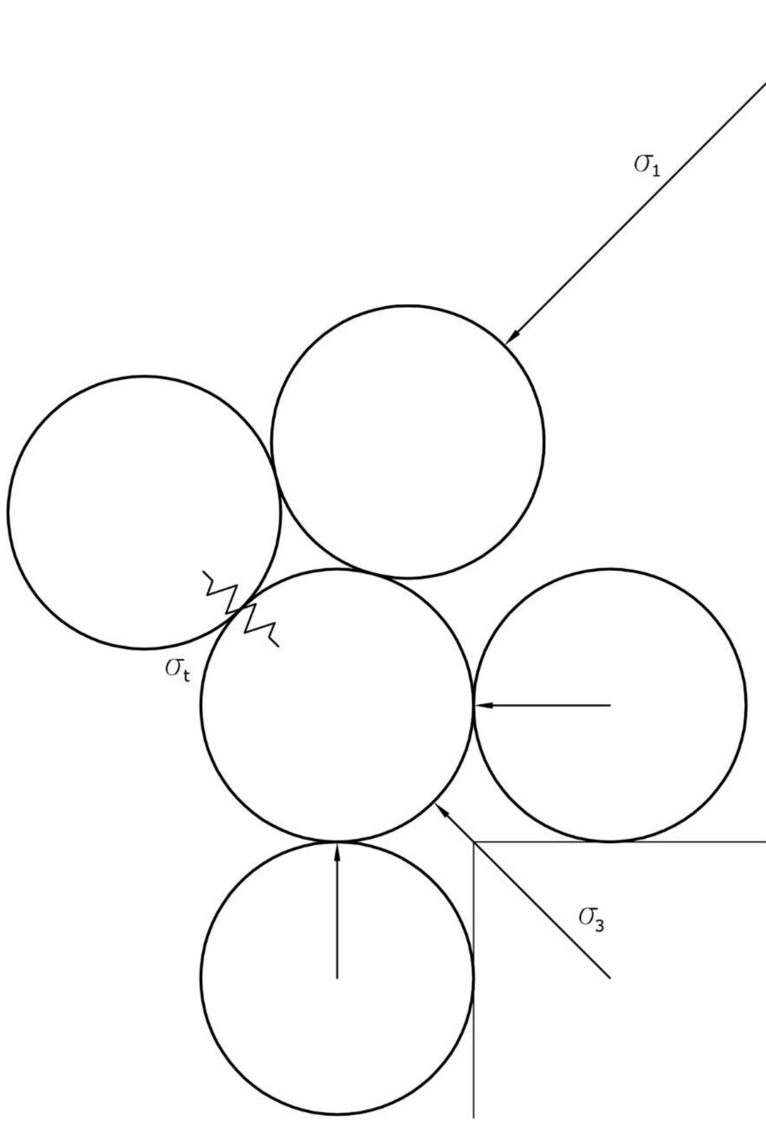

Figure 11. Influence of corner geometry on induced principal stresses

In the given example:

$$\sigma_t = 0{,}14 \cdot 23{,}33 - 7{,}55 = -6{,}96MPa$$

$$\sigma_t = 3{,}26 - 7{,}55 = -6{,}96MPa$$

The component preventing tension is greater than the one inducing it, so no tensile stress will occur in the corner. In the analyzed opening, failure will not initiate in the corners, despite the high stress values—it will occur in the middle of the roof, assuming the tensile strength of the rock is less than 2.32 MPa.

The problem is not in the corners—the problematic areas are the straight segments of the wall. The longer the straight segment, the less stable it is.

Underground openings often have square (rectangular) cross-sections. If the opening is designed as a polygon, for the same width (diameter), the length of the straight segments is reduced, the corner angles increase, and the overall stability becomes more uniform in both corners and straight walls. Naturally, the most favorable shape is a polygon that approximates a circle.

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