Hooke’s Law and Young’s Modulus

When a cylindrical rock specimen is compressed in a press, during a uniaxial compression test, it shortens — its height decreases — and deformation work is performed. The force of the piston pressing on the rock specimen causes a rearrangement of rock particles, as shown in Figure 1.

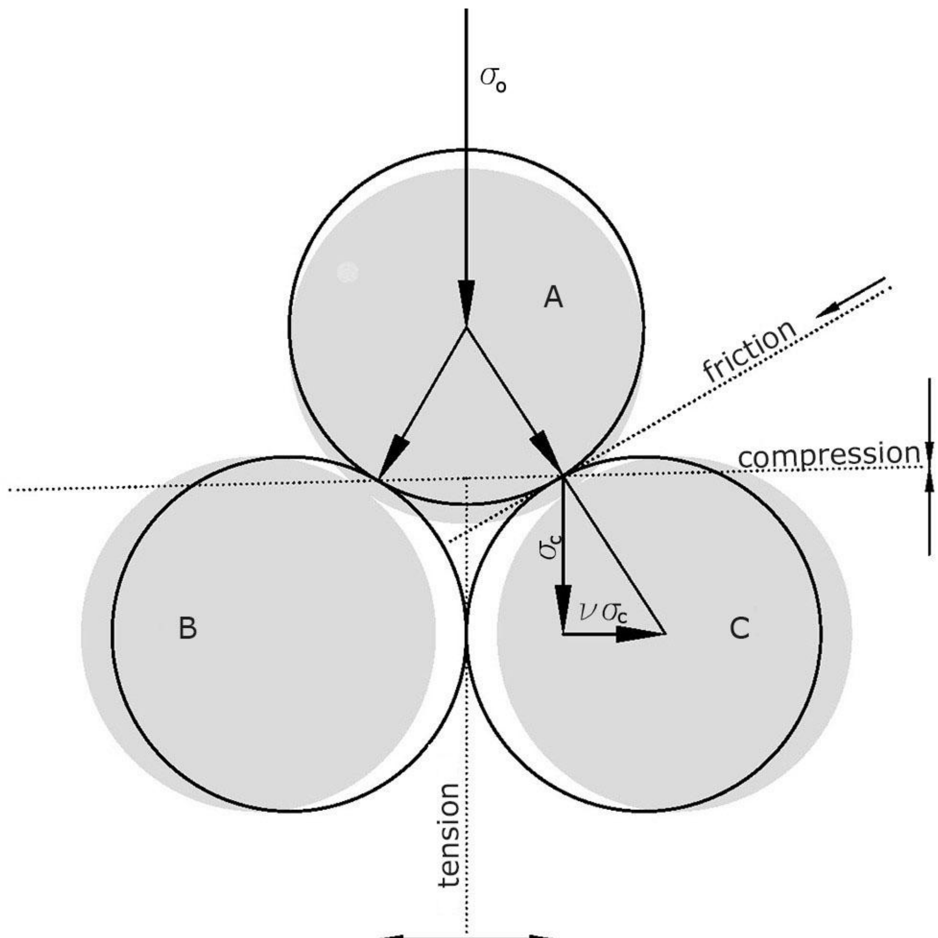

Figure 1. Rearrangement of particles under pressure

In Figure 1, in a symbolic 2D representation, the particles are shown as rigid (non-deformable), which is, of course, not true. According to Hooke’s law, in the plane perpendicular to the direction of loading (σ_0), normal compressive stress (σ_c) is induced and is given by:

$$\sigma_c = E \cdot \varepsilon$$

In reality, during the loading process, the deformation work — that is, the kinetic energy of moving particles — is converted into potential energy of elastically deformed particles, or stress, that is:

$$MNm/_{m^3} = MNm/_{m^3} \cdot m/_m$$

Stress is the potential energy of an elastically deformed unit volume of material, and the modulus of elasticity is the elastic potential — the maximum possible potential energy per unit volume of material. Strain (ε) represents the unit deformation of the material and is the measure of stress.

As seen in Figure 1, and as previously explained, compressed particles move apart, and normal tensile stress is induced between them. During motion, there is frictional resistance between particles, which is proportional to the internal friction angle and the normal force, in this case (σ_t).

Thus, in the uniaxial compression test, the total applied load (σ_0) is distributed into normal compressive stress (σ_c), normal tensile stress (σ_t) and entropy — i.e. the mechanical work of friction that converts into heat ($\sigma_t \, tg\varphi$):

$$\sigma_{\mathrm{o}} = \sigma_c + \sigma_t + \sigma_t \, tg\varphi$$

The total potential energy (stress) of the loaded material consists of compressive stress in the direction of axial strain and tensile stress in the direction of radial (lateral) strain. Compressive stress manifests physically as deformation of the loaded particle, while tensile stress appears as the separation of particles — resisted by the cohesion of the material (tensile strength), described by Hooke’s law.

A material particle consists of subparticles in which the same model is repeated. Therefore, the only stress — the potential energy of elastically deformed material — is tensile stress. What we describe as compressive stress is, in fact, the normal compressive loading of subparticles, resisted by their tensile strength (cohesion) and frictional resistance.

Shear stresses exist only in a continuum — which does not exist.

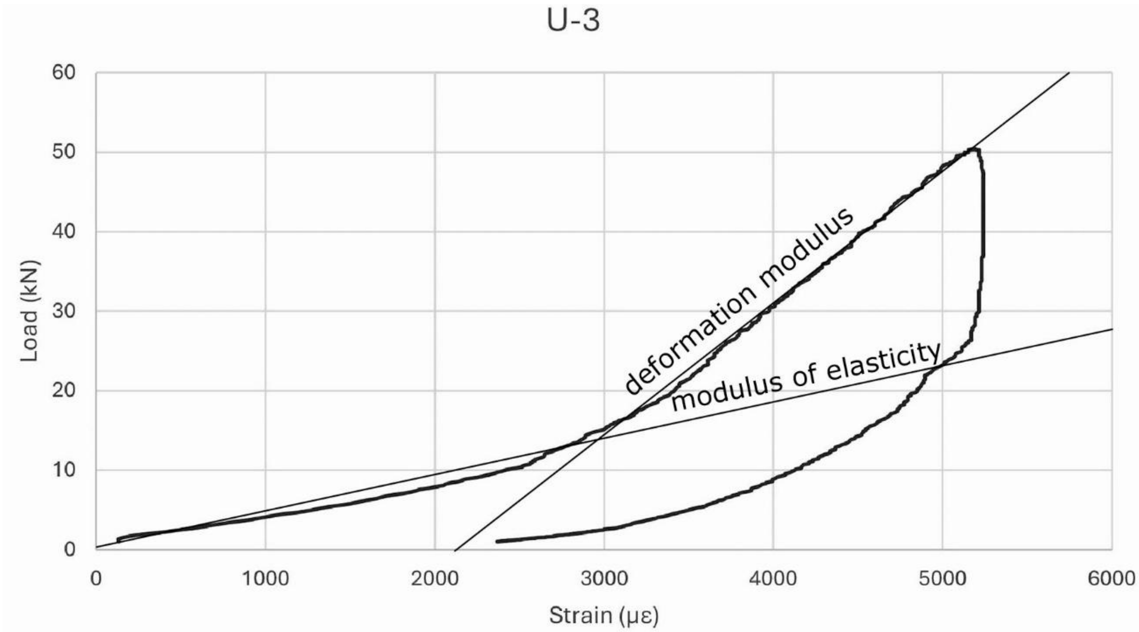

Figure 2. Load and unload diagram

Figure 2 shows a loading and unloading diagram from a uniaxial compression test without lateral support. The diagram is not idealized or adjusted to resemble those in available literature. Lines were added to indicate the moduli. The initial slope is gentle (initial modulus), and this is not — as commonly interpreted — the result of closing existing cracks and fissures. This initial modulus is actually the modulus of elasticity, because only elastic deformation occurs here; particle movements are minimal, so friction is negligible.

The specimen was loaded to approximately half of its compressive strength. Then, unloading was gradually initiated. Reduction of load is not followed by deformation of the specimen. The moment when dilation begins to return is the moment when "compressive stress" in the specimen equals the applied load. Clearly, this point lies on the line that graphically represents the modulus of elasticity.

This is a reliable method for determining the modulus of elasticity.

The difference between the deformation modulus and the modulus of elasticity could be the modulus of entropy.